ESE 2025

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Main Examination

Mechanical Engineering

Topicwise *Conventional* Solved Papers

Paper-I

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Director's Message

In past few years ESE Main exam has evolved as an examination designed to evaluate a candidate's subject knowledge. Studying engineering is one aspect but studying to crack prestigious ESE exam requires altogether different strategy, crystal clear concepts and rigorous practice of previous years' questions. ESE mains being conventional exam has subjective nature of questions, where an aspirant has to write elaborately - step by step with proper and well labeled diagrams and figures. This characteristic of the main exam gave me the aim and purpose to write this book. This book is an effort to cater all the difficulties being faced by students during their preparation right from conceptual clarity to answer writing approach.

MADE EASY Team has put sincere efforts in solving and preparing accurate and detailed explanation for all the previous years' questions in a coherent manner. Due emphasis is made to illustrate the ideal method and procedure of writing subjective answers. All the previous years' questions are segregated subject wise and further they have been categorised topic-wise for easy learning and helping aspirants to solve all previous years' questions of particular area at one place. This feature of the book will also help aspirants to develop understanding of important and frequently asked areas in the exam.

I would like to acknowledge the efforts of entire MADE EASY team who worked hard to solve previous years' questions with accuracy. I hope this book will stand upto the expectations of aspirants and my desire to serve the student community by providing best study material will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group

ESE 2025 Main Examination | Mechanical Engineering *Conventional* Solved Questions | Paper-I

Contents

1 Thermodynamics

Revised Syllabus of ESE: *Thermodynamic systems and processes; properties of pure substance; Zeroth, First and Second Laws of Thermodynamics; Entropy, Irreversibility and availability; analysis of thermodynamic cycles related to energy conversion, ideal and real gases; compressibility factor; Gas mixtures. (Topic of Power Plant: Rankine) (Topics of IC Engine : Otto, Diesel and Dual Cycles).*

1. Basic Concepts, Work and Heat

1.1 An ideal gas is heated at constant volume until its temperature is 3 times the original temperature, then it is expanded isothermally till it reaches its original pressure. The gas is then cooled at constant pressure till it is restored to the original state. Determine the net work done per kg of gas if the initial temperature is 350 K.

[10 marks : 2003]

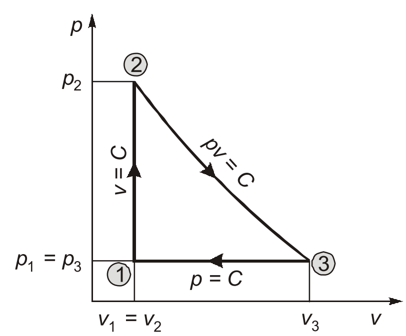
Solution:

The following three processes that form the cycle are shown in p-v diagram.

(i) Process 1-2: Heating at $v = C$

(ii) Process 2-3: Expansion at $T = C$

(iii) Process 3-1: Cooling at $p = C$



Given data: $T_2 = 3T_1$

$T_1 = 350$ K

$\therefore \quad T_2 = 3 \times 350 = 1050$ K

For process 1-2, $\dfrac{p_2}{p_1} = \dfrac{T_2}{T_1} = 3$

Work done per kg of gas: $w_{1\text{-}2} = 0$ $\quad [\because v_1 = v_2]$

For process 2-3, $p_2 v_2 = p_3 v_3$

or $\dfrac{v_2}{v_3} = \dfrac{p_3}{p_2} = \dfrac{p_1}{p_2} = \dfrac{1}{3}$ $\quad [\because p_3 = p_1]$

Work done per kg of gas: $w_{2\text{-}3} = RT_2 \log_e \dfrac{v_3}{v_2} = 0.287 \times 1050 \log_e 3 = 331.06$ kJ/kg

For process 3-1,

Work done per kg of gas: $w_{3\text{-}1} = p_3 (v_1 - v_3) = R(T_1 - T_3)$

$= 0.287\,(350 - 1050) = -200.9$ kJ/kg

Net work done per kg of gas: $w_{net} = w_{1\text{-}2} + w_{2\text{-}3} + w_{3\text{-}1} = 0 + 331.06 - 200.9 = \mathbf{130.16\ kJ/kg}$

1.2 The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as

$$C_p = 2.093 + \frac{41.87}{t+100} \text{ J/°C}$$

where t is the temperature in °C. The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2000 cm^3 to 2400 cm^3 and its temperature increases from 0°C to 100°C.

(i) Find the magnitude of heat interaction.

(ii) How much does the internal energy of the system increase? [5 + 5 = 10 marks : 2009]

Solution:

Given data: V_1 = 2000 cm^3; V_2 = 2400 cm^3; T_1 = 0°C = 273 K; T_2 = 100°C = 373 K

(i) Given $$C_p = 2.093 + \frac{41.87}{t+100} \text{ J/°C}$$

For constant pressure process,

$$\int_1^2 \delta Q = \int_{t_1}^{t_2} C_p dt$$

$$Q_{1-2} = \int_{t_1}^{t_2}\left(2.093+\frac{41.87}{t+100}\right)dt = 2.0936\,(t_2 - t_1) + 41.87 \ln\left(\frac{t_2+100}{t_1+100}\right)$$

$$= 209.3 + 29.02 = \mathbf{238.32\ J}$$

(ii) Work done in the process,

$$W_{1-2} = \int_1^2 pdV = p\int_1^2 dV = p(V_2 - V_1) = 1.01325 \times 10^5 \times (2400 - 2000) \times 10^{-6}$$

$$= 40.53 \text{ J}$$

Change in internal energy,

$$\Delta U = U_2 - U_1 = \Delta Q_{1\text{-}2} - \Delta W_{1\text{-}2} = 238.32 - 40.53 = \mathbf{197.79\ J}$$

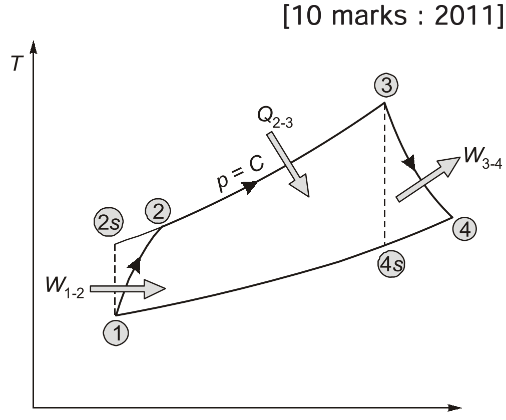
1.3 Obtain an expression for the specific work output of a gas turbine unit in terms of pressure ratio, isentropic efficiencies of the compressor and turbine, and the maximum and minimum temperature, T_3 and T_1. Hence show that the pressure ratio r_p for maximum power is given by

$$r_p = \left[\eta_T \eta_C \frac{T_3}{T_1}\right]^{\gamma/2(\gamma-1)}$$

[10 marks : 2011]

Solution:

In an actual gas turbine, the compressor and the turbine are not isentropic, some losses occur due to internal friction.



Actual cycle without pressure losses

For actual compression process 1-2 in the compressor:

Compressor efficiency: η_c. It is defined as the ratio of isentropic increase in temperature to the actual increase in temperature in the compressor.

Mathematically,

Compressor efficiency: $$\eta_C = \frac{(\Delta T)_{\text{isentropic}}}{(\Delta T)_{\text{actual}}}$$

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \text{...(i)}$$

The compressor efficiency is also called the isentropic efficiency of the compressor.

For isentropic process 1-2s,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{...(ii)}$$

For given values of T_1, p_1 and p_2, we can determine the value of T_{2s}. By substituting the value of T_{2s} in Eq. (i), we can find out the value of T_2 at given value of η_C.

For actual expansion process 3-4 in the turbine:

Turbine efficiency: η_T. It is defined as the ratio of the actual decrease in temperature to the isentropic decrease in temperature in the turbine.

Mathematically,

Turbine efficiency: $$\eta_T = \frac{(\Delta T)_{\text{actual}}}{(\Delta T)_{\text{isentropic}}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad \text{...(iii)}$$

The turbine efficiency is also called the isentropic efficiency of the turbine.

For isentropic process 3-4s,

$$\frac{T_3}{T_{4s}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$T_{4s} = \frac{T_3}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} \quad \text{...(iv)}$$

For given values of T_3, p_1 and p_2, we can determine the value of T_{4s}. By substituting the value of T_{4s} in Eq. (iii), we can find out the value of T_4 at given value of η_T.

Process 2-3: Heat supplied at $p = C$

Heat supplied: $Q_{2\text{-}3} = mc_p\,(T_3 - T_2)$

For unit mass flow rate,

Heat supplied: $q_{2\text{-}3} = c_p\,(T_3 - T_2)$

Actual turbine work output: $W_{3\text{-}4} = W_T = mc_p\,(T_3 - T_4)$

For unit mass flow rate, $w_{3\text{-}4} = w_T = c_p\,(T_3 - T_4)$

Actual compression work required: $W_{1\text{-}2} = W_C = mc_p\,(T_2 - T_1)$

For unit mass flow rate, $w_{1\text{-}2} = w_C = c_p\,(T_2 - T_1)$

Net work output: w_{net} = Turbine work – Compressor work

$$w_{\text{net}} = w_T - w_C$$
$$= c_p\,(T_3 - T_4) - c_p\,(T_2 - T_1)$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \qquad \eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$$

or $$T_3 - T_4 = \eta_T\,(T_3 - T_{4s}) \qquad \text{or} \qquad T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_C}$$

$$\therefore \quad w_{\text{net}} = c_p\eta_T\,(T_3 - T_{4s}) - \frac{c_p}{\eta_C}(T_{2s} - T_1)$$

For ideal process 3-4s, $$\frac{T_3}{T_{4s}} = r_p^{\frac{\gamma-1}{\gamma}} = r_p^z \qquad \left(\because \frac{\gamma-1}{\gamma} = z\right)$$

or $$T_{4s} = \frac{T_3}{r_p^z}$$

$$\mathbf{T_{4s} = T_3 \cdot r_p^{-z}}$$

For ideal process 1-2s, $$\frac{T_{2s}}{T_1} = r_p^{\frac{\gamma-1}{\gamma}} = r_p^z$$

or $$\mathbf{T_{2s} = T_1\, r_p^z}$$

∴ $$w_{net} = c_p\eta_T\left(T_3 - T_3 r_p^{-z}\right) - \frac{c_p}{\eta_C}\left(T_1 r_p^z - T_1\right)$$

For maximum work output condition for the actual cycle,

$$\frac{dw_{net}}{dr_p} = 0$$

$$c_p\,\eta_T\left[0 - T_3(-z)r_p^{-z-1}\right] - \frac{c_p}{\eta_C}\left(T_1\, z r_p^{z-1} - 0\right) = 0$$

or $$\eta_T\, T_3 z r_p^{-z-1} - \frac{T_1 z r_p^{z-1}}{\eta_C} = 0$$

or $$\eta_T T_3 r_p^{-z-1} = \frac{T_1 r_p^{z-1}}{\eta_C}$$

$$\eta_T\,\eta_C \frac{T_3}{T_1} = \frac{r_p^{z-1}}{r_p^{-z-1}} = r_p^{z-1+z+1} = r_p^{2z}$$

or $$\eta_T\,\eta_C\frac{T_3}{T_1} = r_p^{2z}$$

or $$\sqrt{\eta_T\,\eta_C\frac{T_3}{T_1}} = r_p^z$$

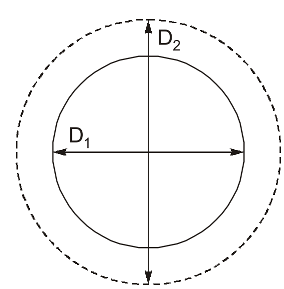
or $$\sqrt{\eta_T\,\eta_C\frac{T_3}{T_1}} = r_p^{\frac{\gamma-1}{\gamma}} \qquad \left(\because z = \frac{\gamma-1}{\gamma}\right)$$

or $$\mathbf{r_p = \left[\eta_T\,\eta_C\frac{T_3}{T_1}\right]^{\frac{\gamma}{2(\gamma-1)}}}$$

1.4 A spherical balloon of 1 m diameter contains a gas at 200 kPa and 300 K. The gas inside the balloon is heated until the pressure reaches 500 kPa. During the process of heating, the pressure is proportional to the diameter of the balloon. Determine the work done by the gas inside the balloon.

[10 marks : 2013]

Solution:



Given data: D_1 = 1 m; p_1 = 200 kPa; T_1 = 300 K; p_2 = 500 kPa

As $$p \propto D$$

∴ $$\frac{p_1}{p_2} = \frac{D_1}{D_2}$$

$$\frac{200}{500} = \frac{D_1}{D_2}$$

or $$D_2 = D_1 \times 2.5 = 1 \times 2.5 = 2.5 \text{ m}$$

$$\frac{p}{D} = \frac{p_1}{D_1} = \frac{p_2}{D_2} = \text{Constant(C)} \qquad \left[C = \frac{p_1}{D_1} = 200 \text{ kPa/m}\right]$$

$$W = \int_1^2 pdV$$

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{8\times 3}\times\pi\left(D^3\right) = \frac{\pi}{6}D^3$$

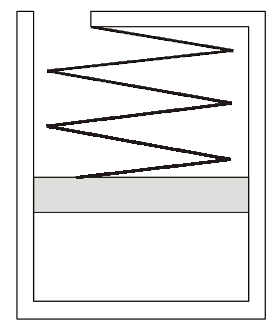
$$\therefore \qquad dV = \frac{3\pi}{6}D^2 dD = \frac{\pi}{2}D^2 dD$$

$$W = \int_1^{2.5}(CD)\times\frac{\pi}{2}D^2 dD = \frac{\pi C}{2}\int_1^{2.5} D^3 dD = \frac{\pi}{2}C\left(\frac{D^4}{4}\right)_1^{2.5}$$

$$W = \frac{3.14}{2}\times\frac{200\times 10^3}{1}\times\frac{\left(2.5^4 - 1^4\right)}{4}$$

$$W = 2988 \times 10^3 \text{ J} = \mathbf{2988 \text{ kJ}}$$

1.5 A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapour water at 120°C, as shown in the figure. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m², and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.



The properties of water are given in the Table below:

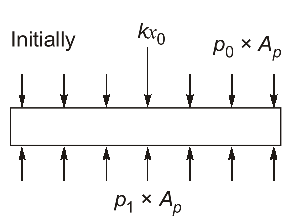
t (°C)	p (kPa)	v_g (m³/kg)	u_g (kJ/kg)	h_g (kJ/kg)
120	198.50 (p_{sat})	0.89186	2529.2	2705.9
151.83	500.00 (p_{sat})	0.37477	2559.5	2746.6
801	500.00	0.99055	3664.2	4159.2
802	500.00	0.99147	3666.1	4161.6
803	500.00	0.99240	3668.0	4163.9
804	500.00	0.99333	3669.9	4166.3
805	500.00	0.99425	3671.8	4168.6

[10 marks : 2014]

Solution:

Let the initial compression in the spring be x_0

Equilibrium equation at state 1



$$p_1 \times A_p = kx_0 + p_0 \times A_p \qquad \text{...(1)}$$

where, p_1 = Saturation pressure at 120°C i.e., 198.50 kPa.

A_p = Area of piston, 0.05 m²

x_0 = Initial compression

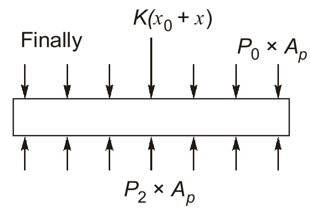
p_0 = Atmospheric pressure

After the heat is transferred in the cylinder, let the piston has moved by a distance x.

Equilibrium equation at state 2

$$p_2 \times A_p = k(x_0 + x) + p_0 \times A_p \qquad ...(2)$$

where, p_2 = Final pressure inside the cylinder, 500 kPa



Subtracting equation (1) from (2)

$$(p_2 - p_1) \times A_p = K \cdot x$$

$$(500 - 198.50) \times 0.05 = 15 \times x$$

$$x = 1.005 \text{ m}$$

At state 1, $m = 0.5$ kg, $u_1 = u_{g@120°C} = 2529.2$ kJ/kg

$$V_1 = v_1 \times m$$

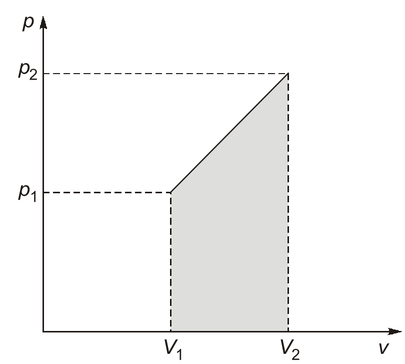
$\because$ $v_1 = v_{g@120°C} = 0.89186 \text{ m}^3/\text{kg}$

$\therefore$ $V_1 = 0.89186 \times 0.5 = 0.44593 \text{ m}^3$

At state 2, $V_2 = V_1 + x \times A_p$

$$= 0.44593 + 1.005 \times 0.05$$

$$= 0.49618 \text{ m}^3$$



$$\text{Specific volume, } v_2 = \frac{V_2}{m} = 0.99236 \text{ m}^3/\text{kg}$$

From given table, at $v = 0.99236 \text{ m}^3/\text{kg}$

$$T_2 = 803°\text{C and } u_2 = 3668.0 \text{ kJ/kg}$$

$$W_{1-2} = \int pdV$$

$$W_{1-2} = \frac{1}{2}(p_1 + p_2)(V_2 - V_1)$$

$$W_{1-2} = \frac{1}{2}(198.5 + 500)(0.49618 - 0.44593) = 17.5498 \text{ kJ}$$

According to 1st law of thermodynamics

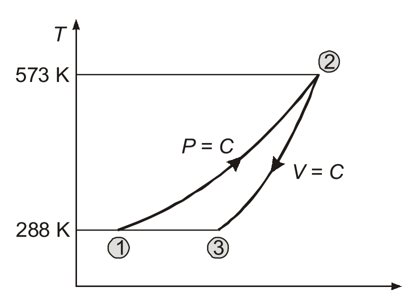
$$Q_{1-2} = U_{1-2} + W_{1-2} = m(u_2 - u_1) + W_{1-2}$$

$$Q_{1-2} = 0.5(3668.0 - 2529.2) + 17.5498$$

$$= 586.9498 \text{ kJ}$$

1.6 1 m^3 of air is heated at constant pressure from 15°C to 300°C and then cooled at constant volume back to its initial temperature. If the initial pressure is 1.03 bar calculate the net heat flow and overall change in entropy. Show the process on a *T-s* diagram. [5 marks : 2014]

Solution:



Given data: $V_1 = 1 \text{ m}^3$; $p_1 = p_2$; $T_1 = 15°\text{C} = (15 + 273) \text{ K} = 288 \text{ K}$

$T_2 = 300°\text{C} = (300 + 273) \text{ K} = 573 \text{ K}$; $p_1 = 1.03 \text{ bar} = 1.03 \times 10^5 \text{ Pa}$

$c_p = 1.005$ kJ/kgK; $c_v = 0.718$ kJ/kgK; $R = 0.287$ kJ/kgK

$$\text{Mass of air: } m = \frac{p_1 V_1}{RT_1} = \frac{1.03 \times 10^5}{287 \times 288} = 1.246 \text{ kg}$$

$$Q = Q_{1\text{-}2} + Q_{2\text{-}3} = mc_p (T_2 - T_1) + mc_v (T_3 - T_2)$$

$$= mc_p (T_2 - T_1) + mc_v (T_1 - T_2) \qquad (\because \; T_1 = T_3)$$
$$= m (T_2 - T_1)(c_p - c_v) = 1.246\,(573 - 288)\,(1.005 - 0.718)$$
$$= \mathbf{102\ kJ}$$

For constant p process,

$$S_2 - S_1 = mc_p \log_e\left(\frac{T_2}{T_1}\right) = 1.246 \times 1.005 \times \log_e\left(\frac{573}{288}\right) = 0.8614\ \text{kJ/K}$$

For constant volume process,

$$S_3 - S_2 = mc_v \log_e\left(\frac{T_3}{T_2}\right) = mc_v \log_e\left(\frac{T_1}{T_2}\right)$$
$$= 1.246 \times 0.718 \times \log_e\left(\frac{288}{573}\right) = -0.6154\ \text{kJ/K}$$

$\therefore$ Overall change of entropy,

$$\Delta S = (S_2 - S_1) + (S_3 - S_2) = 0.8614 - 0.6154 = \mathbf{0.246\ kJ/K}$$

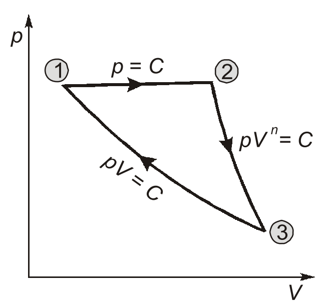
1.7 A certain mass of air is initially at 260°C and 700 kPa and occupies 0.028 m^3. The air is expanded at constant pressure to 0.084 m^3. A polytropic process with n = 1.50 is then carried out, followed by a constant temperature process which completes the cycle. All the processes are reversible processes.

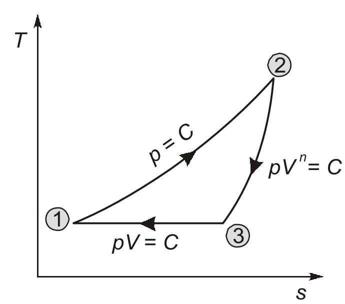
(i) Sketch the cycle on p-v and T-s coordinates and

(ii) Find the efficiency of the cycle. [10 marks : 2015]

Solution:

Given data: T_1 = 260°C = (260 + 273) K = 533 K, p_1 = 700 kPa, V_1 = 0.028 m^3, V_2 = 0.084 m^3





Applying equation of state at state 1,

$$p_1 V_1 = mRT_1$$
$$700 \times 0.028 = m \times 0.287 \times 533$$

or
$$m = 0.1281\ \text{kg}$$

For isobaric process 1-2,

$$\frac{T_2}{V_2} = \frac{T_1}{V_1} \qquad \text{(Charles law)}$$

$$\frac{T_2}{0.084} = \frac{533}{0.028}$$

or
$$T_2 = 1599\ \text{K}$$

For polytropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1} \qquad (\because T_3 = T_1)$$

$$\frac{533}{1599} = \left(\frac{0.084}{V_3}\right)^{1.5-1} = \left(\frac{0.084}{V_3}\right)^{0.5}$$

or $$\left(\frac{533}{1599}\right)^{\frac{1}{0.5}} = \frac{0.084}{V_3}$$

$$\left(\frac{533}{1599}\right)^{2} = \frac{0.084}{V_3}$$

or $$V_3 = 0.756 \text{ m}^3$$

For process 1-2, Work done: $$W_{1\text{-}2} = p_1 (V_2 - V_1)$$
$$= 700 (0.084 - 0.028) = 39.2 \text{ kJ}$$

Heat transfer: $$Q_{1\text{-}2} = m\, c_p (T_2 - T_1)$$
$$= 0.1281 \times 1.005 (1599 - 533) = 137.23 \text{ kJ}$$

For process 2-3, Work done: $$W_{2\text{-}3} = \frac{mR(T_3 - T_2)}{1-n}$$
$$= \frac{0.1281 \times 0.287(533 - 1599)}{1-1.5} = 78.38 \text{ kJ}$$

Heat transfer: $$Q_{2\text{-}3} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W_{2-3} = \left(\frac{1.4 - 1.5}{1.4 - 1}\right) \times 78.38 = -19.59 \text{ kJ}$$

For process 3-1, Work done: $$W_{3\text{-}1} = Q_{3-1} = mRT_1 \log_e \frac{V_1}{V_3}$$
$$= 0.1284 \times 0.287 \times 533 \log_e \frac{0.028}{0.756} = -64.73 \text{ kJ}$$

Net work done: $$W_{net} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$$
$$= 39.2 + 78.38 - 64.73 = 52.85 \text{ kJ}$$

Heat supplied: $$Q_s = Q_{1\text{-}2} = 137.23 \text{ kJ}$$

The efficiency of the cycle, $$\eta = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{W_{net}}{Q_s} = \frac{52.85}{137.23} = 0.3851 = \mathbf{38.51\%}$$

1.8 The pressure in an automobile tire depends on the temperature of the air in the tire. When the air temperature is 25°C, the pressure gauge reads 210 kPa. If the volume of the tire is 0.65 m^3, determine the pressure rise in the tire when the air temperature in the tire rises to 50°C. Also determine the amount of air that must be bleed off to restore pressure to its original value at this temperature. Assume atmospheric pressure to be 100 kPa and R = 0.287 kJ/kgK.

[20 marks : 2017]

Solution:

Given: T_1 = 25°C = 298 K; p_1 = 210 kPa + 100 kPa = 310 kPa; V = 0.65 m^3

p_{atm} = 100 kPa; R = 0.287 kJ/kgK

When T_2 = 50°C, (let p_2 be the pressure inside the tire)

mass of air $$m = \frac{p_1 V}{RT_1} = \frac{310 \times 10^3 \times 0.65}{287 \times 298} = 2.356 \text{ kg}$$

As volume of tire is constant $$p_2 = \frac{mRT_2}{V_2} = \frac{2.356 \times 0.287 \times 10^3 \times (273 + 50)}{0.65}$$

$$p_2 = 336 \text{ kPa}$$

Pressure rise in tire $= p_2 - p_1 = 26$ kPa

To restore pressure to its original value (p_1 = 310 kPa) at T_2 = 50°C, let Δm be the mass of air that must be bleed off.

$$\Delta m = m - m_2$$

$$m_2 = \frac{p_f V}{R T_2}, \text{ where } p_f = p_1$$

$$m_2 = \frac{310 \times 10^3 \times 0.65}{287 \times (273 + 50)} = 2.1736 \text{ kg}$$

and $m = 2.356$ kg

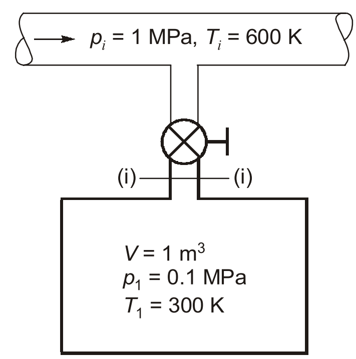
$\therefore$ $\Delta m = 2.356 - 2.1736 = 0.1823$ kg

2. First Law of Thermodynamics

2.1 An insulated tank of 1 m^3 volume contains air at 0.1 MPa and 300 K. The tank is connected to high pressure line in which air at 1 MPa and 600 K flows. The tank is quickly filled with air by opening the valve between the tank and high pressure line. If the air pressure finally in tank is 1 MPa. Determine the mass of air which enters the tank and entropy change associated with filling process.

[20 marks : 2002]

Solution:



Initial condition in the tank,

Given data: $V = 1$ m^3; $p_1 = 0.1$ MPa = 100 kPa; $T_1 = 300$ K

$$p_1 V_1 = m_1 R T_1$$

$$100 \times 1 = m_1 \times 0.287 \times 300$$

or $m_1 = 1.161$ kg

Final condition in the tank, $p_2 = p_i = 1$ MPa = 1000 kPa

Inlet condition, $p_i = 1$ MPa = 1000 kPa

$T_i = 600$ K

Applying unsteady flow energy equation,

$$m_i\left(h_i + \frac{V_i^2}{2} + g z_i\right) + Q - m_e\left(h_e + \frac{V_e^2}{2} + g z_e\right) - W = m_2 u_2 - m_1 u_1$$

where $Q = 0$ insulated tank

$W = 0$ constant volume

Change in K.E. and P.E. are neglected

$m_e = 0$ no exit

$\therefore$ $m_i h_i = m_2 u_2 - m_1 u_1$

where $m_i = m_2 - m_1$

$\therefore$

$$(m_2 - m_1) h_i = m_2 u_2 - m_1 u_1$$

$$m_2 h_i - m_1 h_i = m_2 u_2 - m_1 u_1$$

$$m_2 (h_i - u_2) = m_1 (h_i - u_1)$$

$$m_2 (c_p T_i - c_v T_2) = m_1 (c_p T_i - c_v T_1)$$

$$m_2 c_v \left(\frac{c_p}{c_v} T_i - T_2\right) = m_1 c_v \left(\frac{c_p}{c_v} T_i - T_1\right)$$

or

$$m_2 (\gamma T_i - T_2) = m_1 (\gamma T_i - T_1)$$

$$\frac{p_2 V}{R T_2}(\gamma T_i - T_2) = \frac{p_1 V}{R T_1}(\gamma T_i - T_1)$$

or

$$\frac{p_2}{p_1} \times \frac{T_1}{T_2}(\gamma T_i - T_2) = \gamma T_i - T_1$$

$$\frac{1000}{100} \times \frac{300}{T_2}(1.4 \times 600 - T_2) = 1.4 \times 600 - 300$$

or $252000 - 3000\, T_2 = 540\, T_2$

or $3540\, T_2 = 2520000$

or $T_2 = 711.86$ K

Final mass in the tank: m_2

$$p_2 V = m_2 R T_2$$

$$1000 \times 1 = m_2 \times 0.287 \times 711.86$$

or $m_2 = 4.894$ kg

Mass of air enter the tank: $m_i = m_2 - m_1 = 4.894 - 1.161 =$ **3.733 kg**

due to filling process entropy of mass which was already present in tank also increase.

$$(\Delta S)_{\text{filling}} = (\Delta S)_{\text{entering}} + (\Delta S)_{\text{tank}}$$

$$= m_i\left[c_p \ln\left(\frac{T_2}{T_i}\right) - R\ln\left(\frac{p_2}{p_i}\right)\right] + m_1\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right]$$

$$= 3.733\left[1.005\ln\left(\frac{711.86}{600}\right) - 0.287\ln\left(\frac{1}{1}\right)\right] + 1.161\left[1.005\ln\left(\frac{711.86}{300}\right) - 0.287\ln\left(\frac{1}{0.1}\right)\right]$$

$$= 0.6410 + 0.241 = \mathbf{0.882\ kJ/K}$$

2.2 A large vessel contains steam at a pressure of 20 bar and a temperature of 350°C. This large vessel is connected to a steam turbine through a valve followed by a small initially evacuated tank with a volume of 0.8 m^3. During emergency power requirement, the valve is opened and the tank fills with steam until the pressure is 20 bar. The temperature of the tank is then 400°C. Assume that the filling process takes place adiabatically and the changes in potential and kinetic energies are negligible. By drawing the control volume, calculate the amount of work developed by the turbine in kJ.

[10 marks : 2004]

Solution:

Given data: For large vessel, p = 20 bar, T = 350°C

From superheated steam table,

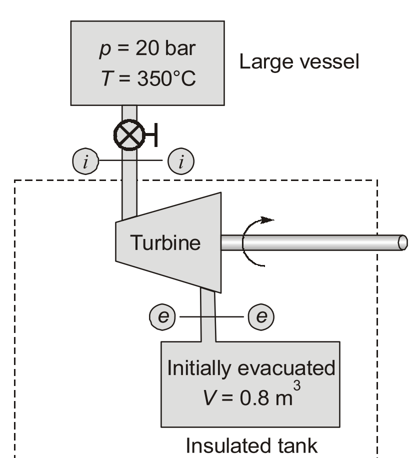
At p = 20 bar, T = 350°C

h_i = 3137 kJ/kg

Tank initially evacuated,

i.e., $m_1 = 0$

$V = 0.8$ m^3



At final state in the tank,

p_2 = 20 bar, T_2 = 400°C

From superheated steam table,

At p_2 = 20 bar, T_2 = 400°C

u_2 = 2945.2 kJ/kg

v_2 = 0.1512 m^3/kg

also $v_2 = \dfrac{V}{m_2}$

$\therefore$ $0.1512 = \dfrac{0.8}{m_2}$

or m_2 = 5.29 kg

Applying unsteady flow energy equation for steam turbine,

$$m_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)+Q-m_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)-W = (m_2u_2-m_1u_1)_{turbine}$$

where $Q = 0$ adiabatic flow

Change in K.E. and P.E. are negligible

No initial and final steam in the turbine

i.e., $m_1 = m_2 = 0$

$\therefore$ $m_i h_i = m_e h_e + W$

or $W = (m_i h_i - m_e h_e)_{turbine}$ ...(i)

Applying unsteady flow energy equation for insulated tank,

$$m_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)+Q-m_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)+W = (m_2 u_2 - m_1 u_1)$$

where $Q = 0$ [adiabatic process]

$W = 0$, [$\because V = C$]

Change in K.E. and P.E. are negligible

$m_e = 0$ (no exit)

$m_1 = 0$ (initially tank evacuated)

$\therefore$ $(m_i h_i)_{Tank} = m_2 u_2$

where energy outlet of the turbine = energy inlet of the tank

$\therefore$ $m_e h_e = m_2 u_2$

Substituting $m_e h_e = m_2 u_2$ in Eq. (i), we get

$W = m_i h_i - m_2 u_2 = m_2 (h_i - u_2)$ $\because m_i = m_2$

$= 5.29\,(3137 - 2945.2) = \mathbf{1014.62\ kJ}$

2.3 An air compressor is used to fill rapidly a 3 m^3 tank at 20°C and 1 atm. The filling process is governed by the law $pv^{1.4}$ = constant. The kinetic and potential energy effects are negligible. The ratio of the final to the initial mass of air in the tank is 4. Work out the following:

(i) Sketch the system and list the assumptions made

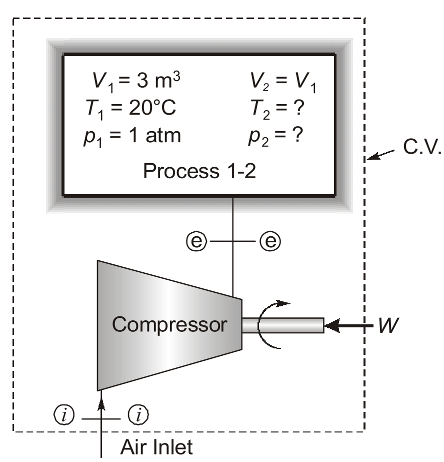
(ii) Work input to the compressor. [15 marks : 2007]

Solution:

Given data: Initial condition in tank: $V_1 = 3$ m^3, $T_1 = 20°C = (20 + 273)$ K = 293 K

$p_1 = 1$ atm = 101.325 kPa, Filling process is governed by law $pv^{1.4}$ = constant

Final mass of air in tank : $m_2 = 4m_1$



Assumptions:

1. Adiabatic process: $Q = 0$
2. Change in kinetic and potential energy are negligible

$p_1V_1 = m_1RT_1$

$101.325 \times 3 = m_1 \times 0.287 \times 293$

or $m_1 = 3.6148$ kg

$\therefore$ $m_2 = 4m_1 = 4 \times 3.566 = 14.459$ kg

$pv = mRT$

$\frac{p}{mT} = \frac{R}{v} = C$, R and V are constant.

For process 1-2, $\frac{p_1}{m_1T_1} = \frac{p_2}{m_2T_2}$

$$\frac{101.325}{m_1\times 293} = \frac{p_2}{4\,m_1\,T_2}$$

or $p_2 = 1.38\, T_2$

For adiabatic filling tank process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{293} = \left(\frac{1.38\,T_2}{101.325}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{1.38}{101.325}\right)^{0.2857} \times T_2^{0.2857}$$

or $$\frac{T_2}{T_2^{0.2857}} = 293\times\left(\frac{1.38}{101.325}\right)^{0.2857} = 85.86$$

or $$T_2^{0.7143} = 85.86$$

or $$T_2 = (85.86)^{\frac{1}{0.7143}} = 509.62 \text{ K}$$

Mass conservation, $$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e \qquad (\because \text{ no exit of mass } \dot{m}_e = 0)$$

$$m_2 - m_1 = \dot{m}_i$$

and $$U_2 - U_1 = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - W_{c.v.}$$

No heat transfer $$\dot{Q} = 0, \quad \dot{m}_e = 0$$

$\therefore$ $$m_2\, c_v\, T_2 - m_1\, c_v\, T_1 = \dot{m}_i h_i - W_{c.v.}$$

$$14.459 \times 0.718 \times 510.11 - 3.6148 \times 0.718 \times 293 - 10.8445 \times 1.005 \times 293 = -W_{cv}$$

$$W_{cv} = \mathbf{-1341.9529 \text{ kW}}$$

Negative sign indicates work supplied.

2.4 Steam enters a turbine at an enthalpy of 3300 kJ/kg and a velocity of 180 m/s. The steam comes out of turbine at an enthalpy of 2700 kJ/kg with a velocity of 120 m/s. At the condition of steady state, the turbine develops work equal to 550 kJ/kg of steam flowing through the turbine. The heat transfer between the turbine and its surroundings occurs at an average temperature of 370 K. The entropy of steam at inlet and exit of turbine are 6.932 kJ/kgK and 7.361 kJ/kgK, respectively. Neglecting the changes in potential energy between inlet and outlet, work out the following:

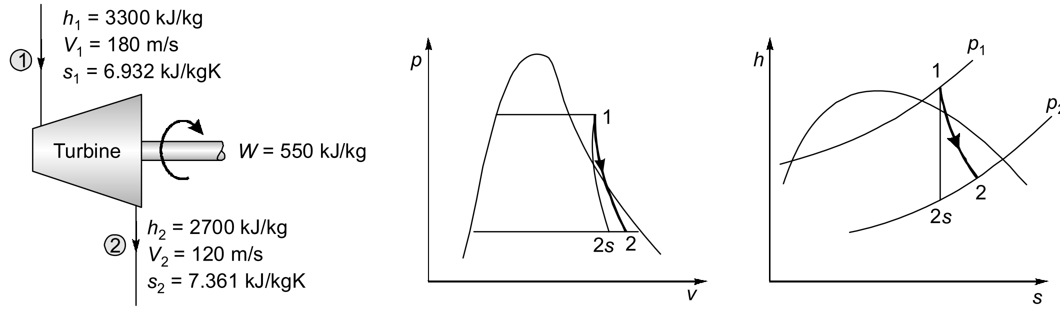
(i) Sketch the system and show the process on *p-v* and *h-s* diagrams.

(ii) Calculate the rate at which entropy is produced within the turbine per kg of steam flowing.

[10 marks : 2007]

Solution:

Let us take entering state as (1) and exit state as (2)



(a) Steam turbine **(b) p-v diagram** **(c) h-s diagram**

At inlet state-1,

$$h_1 = 3300 \text{ kJ/kg}$$
$$V_1 = 180 \text{ m/s}$$
$$s_1 = 6.932 \text{ kJ/kgK}$$

At outlet state-2,

$$h_2 = 2700 \text{ kJ/kg}$$
$$V_2 = 120 \text{ m/s}$$
$$s_2 = 7.361 \text{ kJ/kgK}$$

Applying energy equation for steady flow

$$q + \left(h_1 + \frac{V_1^2}{2}\right) = w + \left(h_2 + \frac{V_2^2}{2}\right)$$

Note: Make all terms in kJ/kg

$$q + \left(h_1 + \frac{V_1^2}{2000}\right) = w + \left(h_2 + \frac{V_2^2}{2000}\right)$$

where q, w, h_1, h_2 are in kJ/kg
and V_1 and V_2 are in m/s

or
$$q = w + (h_2 - h_1) + \left(\frac{V_2^2}{2000} - \frac{V_1^2}{2000}\right)$$
$$= 550 + \left[(2700 - 3300) + \left(\frac{120^2}{2000} - \frac{180^2}{2000}\right)\right]$$
$$= 550 + [-609] = -59 \text{ kJ/kg}$$

Applying second law to the steady flow

$$\frac{dS}{dt} = \frac{Q}{T} + \Sigma m_i s_i - \Sigma m_0 s_0 + S_{gen}$$

As
$$\frac{dS}{dt} = 0 \text{ (steady flow)}$$

Entropy generated per second,

$$S_{gen} = ms_2 - ms_1 - \frac{Q}{T} = m\left(s_2 - s_1 - \frac{Q}{mT}\right)$$
$$\frac{S_{gen}}{m} = s_2 - s_1 - \frac{q}{T}$$
$$s_{gen} = s_2 - s_1 - \frac{q}{T} = (7.361 - 6.932) - \left(-\frac{59}{370}\right) = 0.429 + 0.1595$$
$$= \mathbf{0.588 \text{ kJ/kgK}}$$

2.5 A tank contains 50 kg of water initially at a temperature of 30°C. Water at the rate of 200 kg/h and temperature of 30°C enters the tank through an inlet pipe. A cooling coil immersed in the tank removes heat energy from water at the rate of 8 kW. A mechanical stirrer ensures thorough mixing of water in the tank so as to maintain a uniform temperature of water at any instant and in the process add heat energy at the rate of 0.2 kW to water. Neglecting kinetic and potential energy changes and taking the average specific heat of water as 4.2 kJ/kgK, derive an expression for the variation of instantaneous temperature of water in the tank with respect to time.

[10 marks : 2008]

Solution:

For given control volume, mass conservation

$$\left(\frac{d\dot{m}}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

Since mass leaving control volume $\dot{m}_e = 0$

$$\left(\frac{d\dot{m}}{dt}\right)_{cv} = \dot{m}_i$$

$$m_2 - m_1 = m_i \quad \text{... (i)}$$

Applying energy balance for control volume neglecting kinetic and potential energy changes.

$$\left(\frac{du}{dt}\right)_{c.v.} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv} \quad \text{... (ii)}$$

Consider after time t temperature of mass in control volume is T

Water enter into control volume after time t is $m_i = 200t$

$\therefore$ from equation (i) $\quad m_2 = m_1 + 200t = 50 + 200t$

$\therefore$ equation (ii) becomes $\quad m_2 u_2 - m_1 u_1 = \int(\dot{m}_i c T_i + \dot{Q} - \dot{W}_{cv})dt = m_i c T_i + Q - W_{cv}$

$$(50 + 200t)\, 4.2T - 50 \times 4.2 \times 303 = (200t)\, 4.2 \times 303 + (0.2t)\, 3600 - (8t)\, 3600$$

$$50\,(1 + 4t)\, 4.2T = 226440t + 50 \times 4.2 \times 303$$

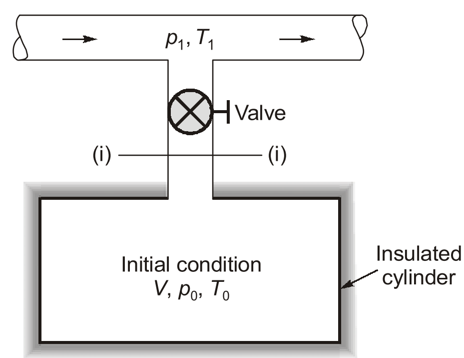
$$T = \frac{1078.28t + 303}{1 + 4t}$$

2.6 A pressure cylinder of volume V contains air at pressure p_0 and temperature T_0. It is to be filled from a compressed air line maintained at constant pressure p_1 and temperature T_1. Show that the temperature of air in the cylinder after it has been charged to the pressure of the line is given by

$$T = \frac{\gamma T_1}{1 + \dfrac{p_0}{p_1}\left[\gamma \dfrac{T_1}{T_0} - 1\right]}$$

[10 marks : 2011]

Solution:



T_0 = Initial temperature in a cylinder

p_0 = Initial pressure in a cylinder

V = Volume of air in a cylinder

p_1 = Pressure of air in pipeline

T_1 = Temperature of air in pipeline

T = Temperature of air after cylinder charged

p_2 = Pressure of air after cylinder charged = p_1

Applying unsteady flow energy equation,

$$m_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - m_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) + Q - W = mu - m_0 u_0 \quad \text{...(1)}$$

where $\quad Q = 0$ (Insulated cylinder)

$\quad W = 0$ (Constant volume cylinder)

Change in K.E. and P.E. are neglected

$\quad m_e = 0$ (No exit)

$\therefore$ Equation (1) becomes, $\quad m_i h_i = mu - m_0 u_0$

where $\quad m_i = m - m_0$

$\therefore$

$$(m - m_0)\, h_i = mu - m_0 u_0$$

$$mh_i - m_0\, h_i = mu - m_0 u_0$$

$$m\,(h_i - u) = m_0\,(h_i - u_0)$$

$$m\,(c_p T_i - c_v T) = m_0\,(c_p\, T_i - c_v T_0)$$

$$mc_v\left(\frac{c_p}{c_v} T_i - T\right) = m_0 c_v\left(\frac{c_p}{c_v} T_i - T_0\right)$$

$$m\,(\gamma T_i - T) = m_0\,(\gamma T_i - T_0))$$